11.6.3 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 11 materials.

Graph the following equations.

For help with these exercises, click the resource below:

- The general form of a conic section

1. $x^2 + 2xy + y^2 - x\sqrt{2} + y\sqrt{2} - 6 = 0$
2. $7x^2 - 4xy\sqrt{3} + 3y^2 - 2x - 2y\sqrt{3} - 5 = 0$
3. $5x^2 + 6xy + 5y^2 - 4\sqrt{2}x + 4\sqrt{2}y = 0$
4. $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y - 16 = 0$
5. $13x^2 - 34xy\sqrt{3} + 47y^2 - 64 = 0$
6. $x^2 - 2\sqrt{3}xy - y^2 + 8 = 0$
7. $x^2 - 4xy + 4y^2 - 2x\sqrt{5} - y\sqrt{5} = 0$
8. $8x^2 + 12xy + 17y^2 - 20 = 0$

Graph the following equations.

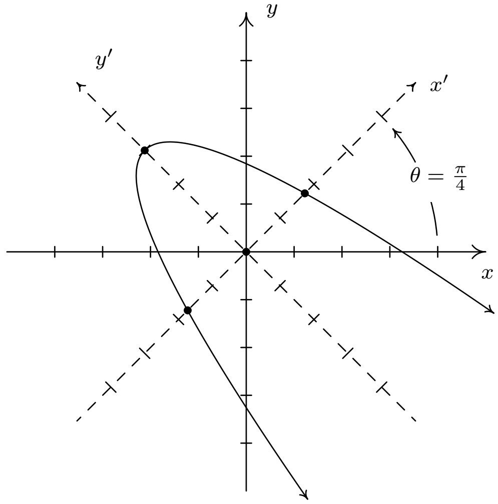
9. $r = \dfrac{2}{1 - \cos(\theta)}$
10. $r = \dfrac{3}{2 + \sin(\theta)}$
11. $r = \dfrac{3}{2 - \cos(\theta)}$
12. $r = \dfrac{2}{1 + \sin(\theta)}$
13. $r = \dfrac{4}{1 + 3\cos(\theta)}$
14. $r = \dfrac{2}{1 - 2\sin(\theta)}$
15. $r = \dfrac{2}{1 + \sin(\theta - \frac{\pi}{3})}$
16. $r = \dfrac{6}{3 - \cos\left(\theta + \frac{\pi}{4}\right)}$

The matrix $A(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ is called a **rotation matrix**. We've seen this matrix most recently in the proof of used in the proof of Theorem 11.9.

17. Show the matrix from Example 8.3.3 in Section 8.3 is none other than $A\left(\frac{\pi}{4}\right)$.
18. Discuss with your classmates how to use $A(\theta)$ to rotate points in the plane.
19. Using the even / odd identities for cosine and sine, show $A(\theta)^{-1} = A(-\theta)$. Interpret this geometrically.

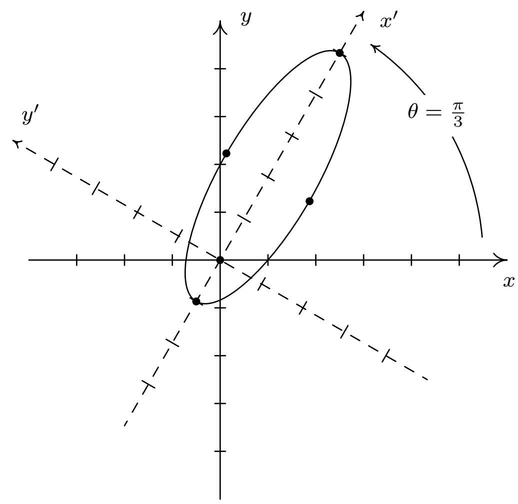
11.6.4 Answers

1. $x^2 + 2xy + y^2 - x\sqrt{2} + y\sqrt{2} - 6 = 0$ becomes $(x')^2 = -(y' - 3)$ after rotating counter-clockwise through $\theta = \frac{\pi}{4}$.



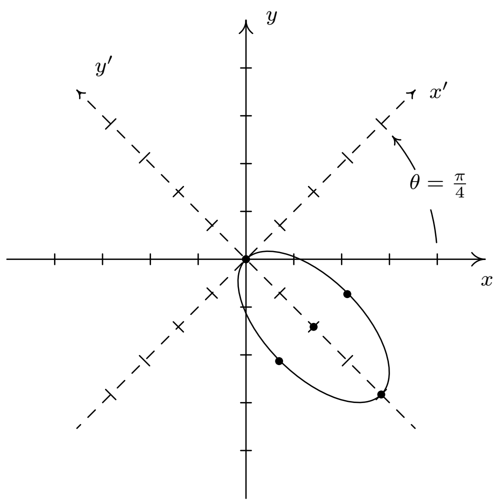
$x^2 + 2xy + y^2 - x\sqrt{2} + y\sqrt{2} - 6 = 0$

2. $7x^2 - 4xy\sqrt{3} + 3y^2 - 2x - 2y\sqrt{3} - 5 = 0$ becomes $\frac{(x'-2)^2}{9} + (y')^2 = 1$ after rotating counter-clockwise through $\theta = \frac{\pi}{3}$



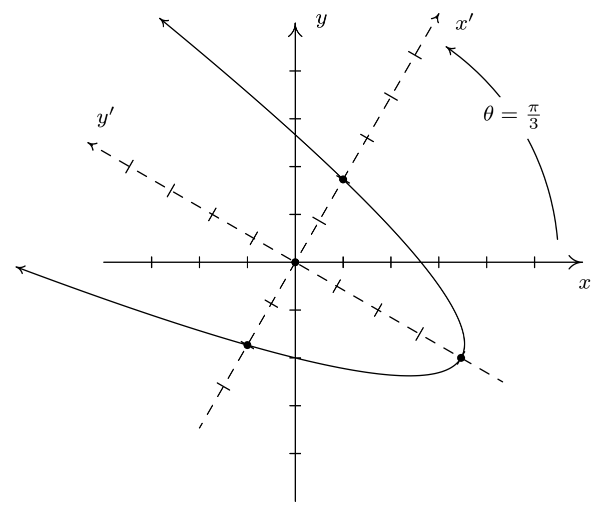
$7x^2 - 4xy\sqrt{3} + 3y^2 - 2x - 2y\sqrt{3} - 5 = 0$

3. $5x^2 + 6xy + 5y^2 - 4\sqrt{2}x + 4\sqrt{2}y = 0$ becomes $(x')^2 + \frac{(y'+2)^2}{4} = 1$ after rotating counter-clockwise through $\theta = \frac{\pi}{4}$.



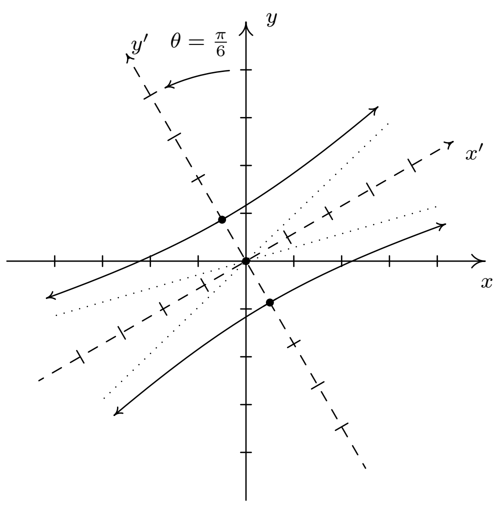
$5x^2 + 6xy + 5y^2 - 4\sqrt{2}x + 4\sqrt{2}y = 0$

4. $x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y - 16 = 0$ becomes$(x')^2 = y' + 4$ after rotating counter-clockwise through $\theta = \frac{\pi}{3}$



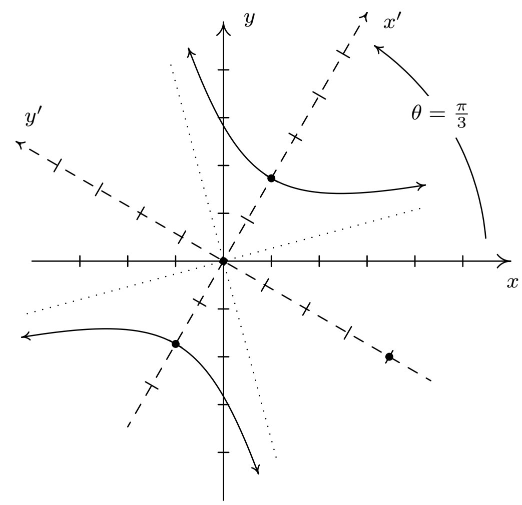
$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y - 16 = 0$

5. $13x^2 - 34xy\sqrt{3} + 47y^2 - 64 = 0$ becomes $(y')^2 - \frac{(x')^2}{16} = 1$ after rotating counter-clockwise through $\theta = \frac{\pi}{6}$.



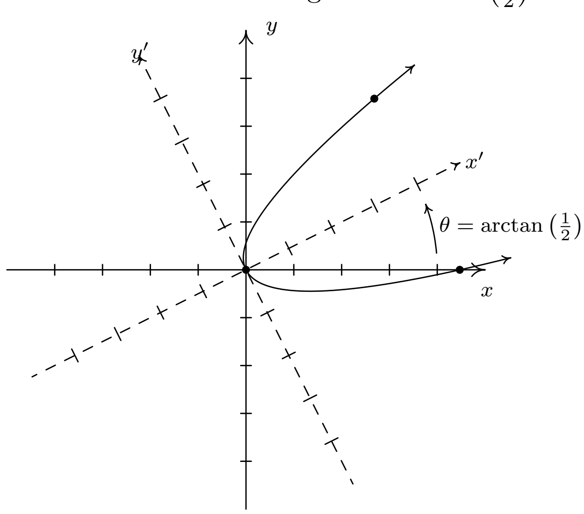
$13x^2 - 34xy\sqrt{3} + 47y^2 - 64 = 0$

6. $x^2 - 2\sqrt{3}xy - y^2 + 8 = 0$ becomes $\frac{(x')^2}{4} - \frac{(y')^2}{4} = 1$ after rotating counter-clockwise through $\theta = \frac{\pi}{3}$



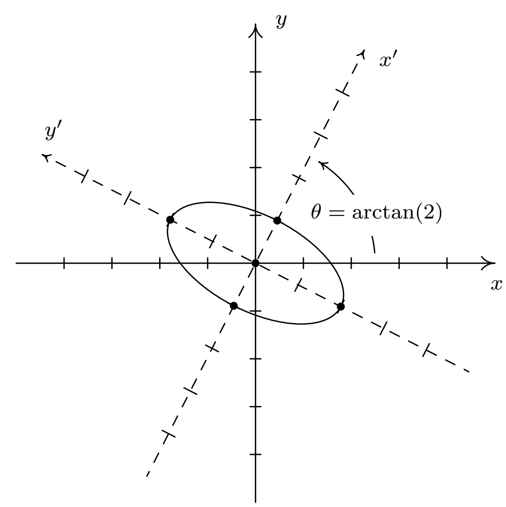
$x^2 - 2\sqrt{3}xy - y^2 + 8 = 0$

7. $x^2 - 4xy + 4y^2 - 2x\sqrt{5} - y\sqrt{5} = 0$ becomes $(y')^2 = x$ after rotating counter-clockwise through $\theta = \arctan\left(\frac{1}{2}\right)$.



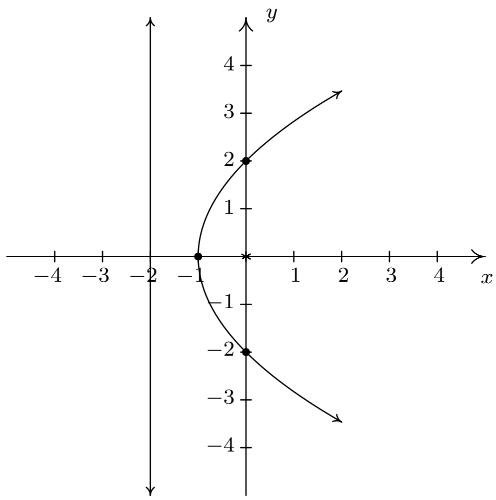
$x^2 - 4xy + 4y^2 - 2x\sqrt{5} - y\sqrt{5} = 0$

8. $8x^2 + 12xy + 17y^2 - 20 = 0$ becomes $(x')^2 + \frac{(y')^2}{4} = 1$ after rotating counter-clockwise through $\theta = \arctan(2)$

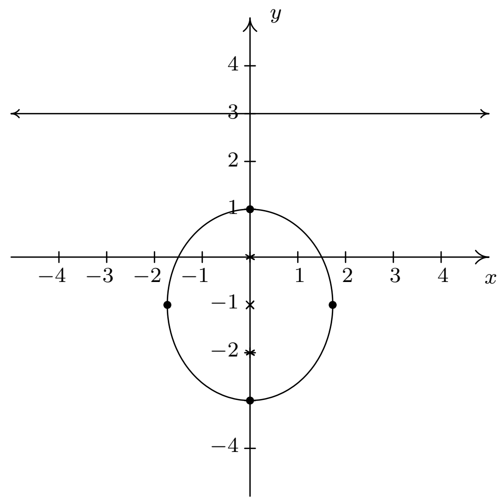


$8x^2 + 12xy + 17y^2 - 20 = 0$

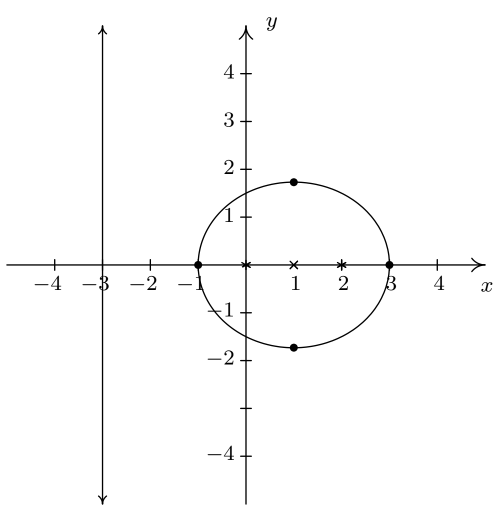
9. $r = \frac{2}{1-\cos(\theta)}$ is a parabola
directrix $x = -2$, vertex $(-1, 0)$
focus $(0, 0)$, focal diameter 4



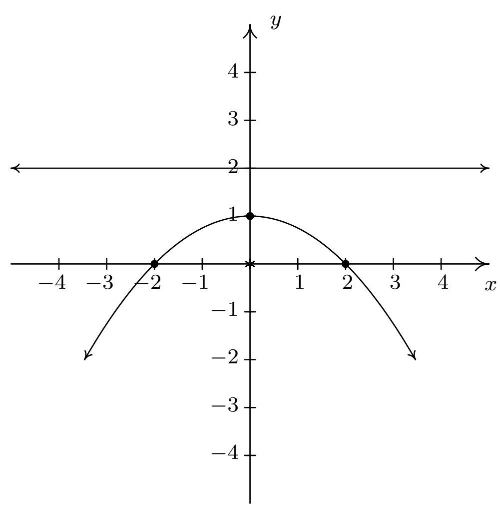
10. $r = \frac{3}{2+\sin(\theta)} = \frac{\frac{3}{2}}{1+\frac{1}{2}\sin(\theta)}$ is an ellipse
directrix $y = 3$, vertices $(0, 1)$, $(0, -3)$
center $(0, -2)$, foci $(0, 0)$, $(0, -2)$
minor axis length $2\sqrt{3}$



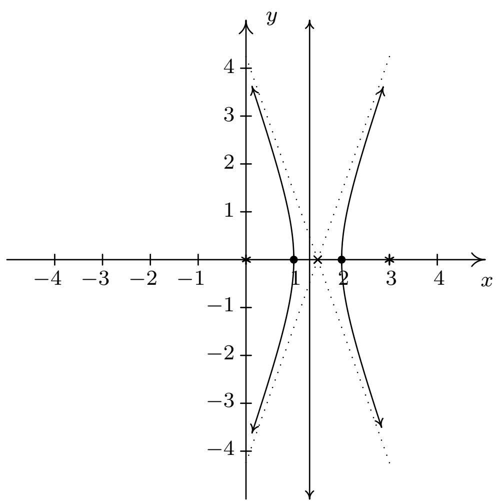
11. $r = \frac{3}{2-\cos(\theta)} = \frac{\frac{3}{2}}{1-\frac{1}{2}\cos(\theta)}$ is an ellipse
directrix $x = -3$, vertices $(-1, 0)$, $(3, 0)$
center $(1, 0)$, foci $(0, 0)$, $(2, 0)$
minor axis length $2\sqrt{3}$



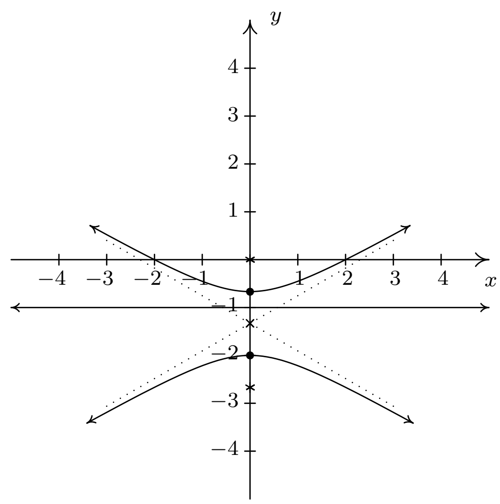
12. $r = \frac{2}{1+\sin(\theta)}$ is a parabola
directrix $y = 2$, vertex $(0, 1)$
focus $(0, 0)$, focal diameter 4



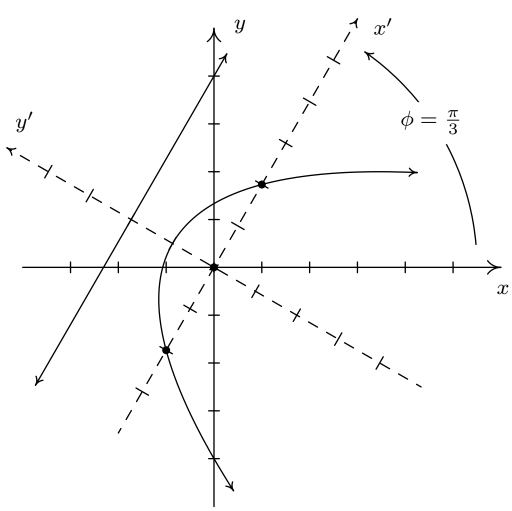
13. $r = \frac{4}{1+3\cos(\theta)}$ is a hyperbola
directrix $x = \frac{4}{3}$, vertices $(1,0)$, $(2,0)$
center $\left(\frac{3}{2},0\right)$, foci $(0,0)$, $(3,0)$
conjugate axis length $2\sqrt{2}$



14. $r = \frac{2}{1-2\sin(\theta)}$ is a hyperbola
directrix $y=-1$, vertices $\left(0,-\frac{2}{3}\right)$, $(0,-2)$
center $\left(0,-\frac{4}{3}\right)$, foci $(0,0)$, $\left(0,-\frac{8}{3}\right)$
conjugate axis length $\frac{2\sqrt{3}}{3}$



15. $r = \frac{2}{1+\sin\left(\theta-\frac{\pi}{3}\right)}$ is
the parabola $r = \frac{2}{1+\sin(\theta)}$
rotated through $\phi = \frac{\pi}{3}$



16. $r = \frac{6}{3-\cos\left(\theta+\frac{\pi}{4}\right)}$ is the ellipse
$r = \frac{6}{3-\cos(\theta)} = \frac{2}{1-\frac{1}{3}\cos(\theta)}$
rotated through $\phi = -\frac{\pi}{4}$